Northern Illinois University

Studies of Conventional and ERL-Based Re-circulator Electron Cooling for an Electron Ion Collider

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DOE-NP Accelerator R&D PI Meeting
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Project Description, Current Status and Main Goals

- **Description:** electron cooling code development and simulations.
- **Current Status:** Simulation studies of:
  - Bunched electron beam cooling of low energy ion beams at IMP.
  - Density modulation in the modulator section of the CeC.
- **Main Goals** for FY 2018-19 FOA (the award is not part of the Lab Base):
  - Proof of principle **demonstration of electron cooling** with the **first particle-level detailed** simulations (Jones Report:: line 3: High, A).
  - **Develop a high-performance code** with these capabilities, and other beam dynamics challenges beyond cooling (Jones Report:: line 4: High, A).
  - **Applications** to electron-ion collider (JLEIC) modeling, design, and optimization (Jones Report:: line 39: High).

Expenditures and Milestones

	FY10+FY11	FY12+FY13	FY14+FY15	FY16+FY17	FY18+FY19	TOTALS
Funds Allocated	0+56	55+52=107	50+54=104	50+50=100	55+55=110	$477K
Actual Costs to Date	56	107	104	100	110	$477K

	FY18	FY19
Quarter 1	Identification of speed and parallel efficiency bottlenecks in the current version of PHAD.	Re-establish closer working relationship with JLEIC project management.
Quarter 2	Amelioration of speed and efficiency bottlenecks in PHAD.	Update our understanding to enhance responsiveness to the progressing needs of electron-ion collider cooling needs.
Quarter 3	DC electron cooling initial rate estimations for different ion species (charge states) at low energy.	Provide code and assistance as needed to JLAB staff to simulate electron cooling.
Quarter 4	Low energy electron cooling initial rate estimations for different initial distributions and external fields.	Initiate possible collaboration with BNL EIC project.

Outline

- Electron cooling and the friction force
  - Importance of collisions
- Collisional methods for electron cooling
  - Challenges
  - PHAD overview and updates
- PHAD benchmarking with IMP experiments
- Simulations updates
- Summary

Electron Cooling and Friction Force

- **Electron cooling:** a method used to significantly reduce the 6D phase space volume of ion beams.
- Ions copropagates with electrons → ions experience a dynamical friction.
- The average rate of the momentum transfer is known as the **dynamical friction force**.
- For an isotropic electron temperature, an analytical formula was first obtained*.
- For years, the use of the classical dynamical friction force formula developed the physical intuition that collisions due to small impact parameters (close encounters) can be ignored.

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* Chandrasekhar S 1942 *Principles of Stellar Dynamics* (Chicago, IL: University of Chicago Press)

* Trubnikov B A 1965 Particle interactions in a fully ionized plasma *Rev. Plasma Phys.* **1** 105

Close Encounters and the Dynamical Friction

Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam

George I. Bell [a,*], David L. Bruhwiler [a], Alexei Fedotov [b], Andrey Sobol [a], Richard S. Busby [a], Peter Stoltz [a], Dan T. Abell [a], Peter Messmer [a], Ilan Ben-Zvi [b], Vladimir Litvinenko [b]

[a] *Tech-X Corporation, Accelerator Technology, 5621 Arapahoe Avenue, Suite A, Boulder, CO 80303, United States*
[b] *Brookhaven National Lab, Upton, NY 11973, United States*

- Discrepancy between direct numerical simulations and tested analytic formulae.

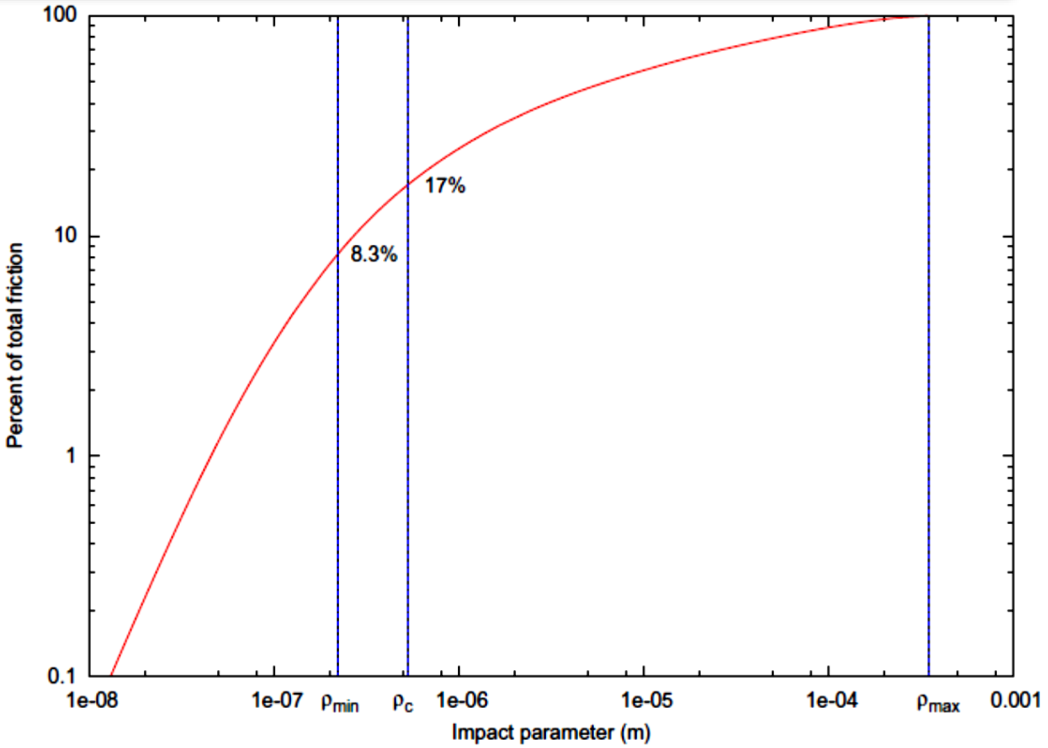
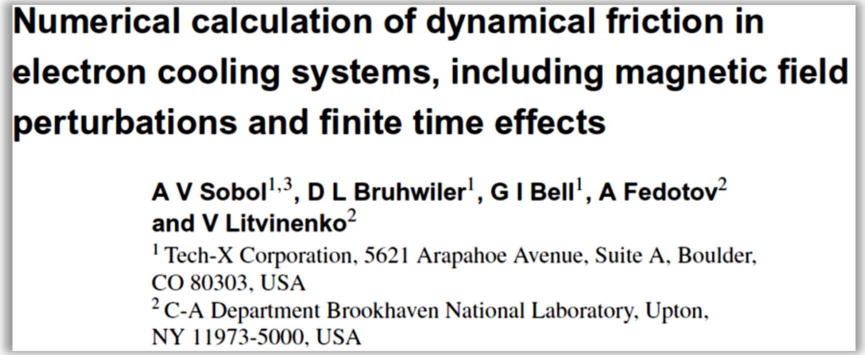


Fig. 6. Cumulative friction by impact parameter. If there are no collisions with impact perimeter less than ρ_c, the dynamical friction will be reduced by 17%.

Bell, G. I., Bruhwiler, D. L., Fedotov, A., Sobol, A., Busby, R. S., Stoltz, P., ... & Litvinenko, V. (2008). Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam. *Journal of Computational Physics*, *227*(19), 8714-8735.

Close Encounters and the Dynamical Friction



Numerical calculation of dynamical friction in electron cooling systems, including magnetic field perturbations and finite time effects

A V Sobol[1,3], D L Bruhwiler[1], G I Bell[1], A Fedotov[2] and V Litvinenko[2]
[1] Tech-X Corporation, 5621 Arapahoe Avenue, Suite A, Boulder, CO 80303, USA
[2] C-A Department Brookhaven National Laboratory, Upton, NY 11973-5000, USA

- The friction force on an ion can significantly deviate from its average due to rare, strong collisions (during a single pass through an electron cooling section).
  - any analytic formula provides a limited view of the full dynamics

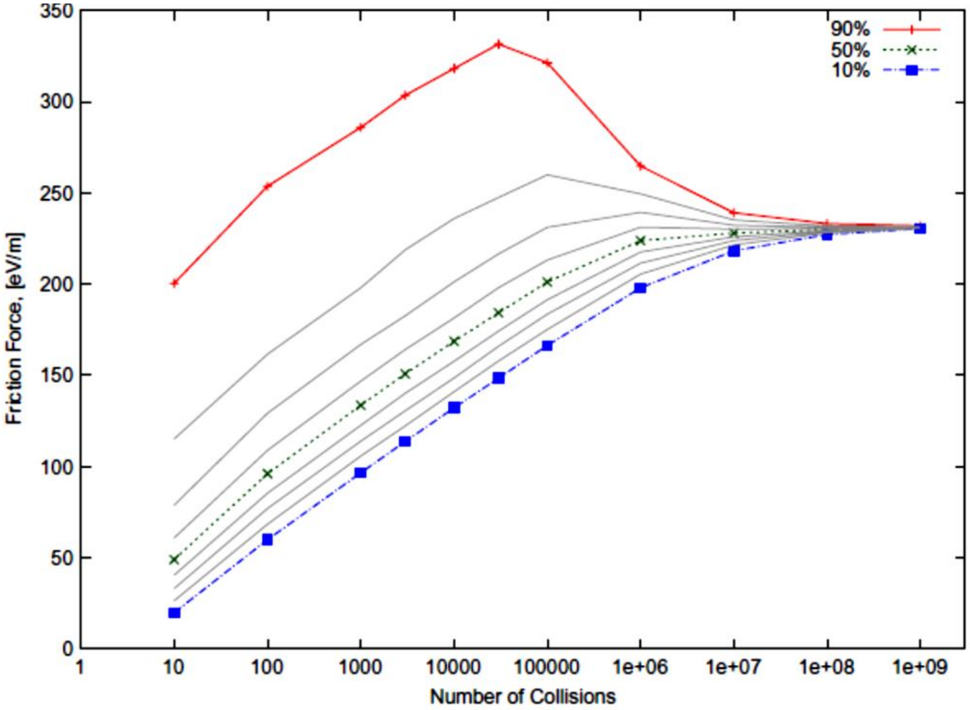


Figure 8. The friction force resulting from a sequence of collisions as an RV. The 10-quantiles (deciles) are plotted as a function of the number of collisions: thick lines are 10%-, 50%- and 90%-quantiles and thin lines are 20%-, 30%-, 40%-, 60%-, 70%- and 80%-quantiles. The 80% confidence interval is very wide unless the number of collisions is extremely large.

Sobol, A. V., Bruhwiler, D. L., Bell, G. I., Fedotov, A., & Litvinenko, V. (2010). Numerical calculation of dynamical friction in electron cooling systems, including magnetic field perturbations and finite time effects. *New Journal of Physics*, *12*(9), 093038.

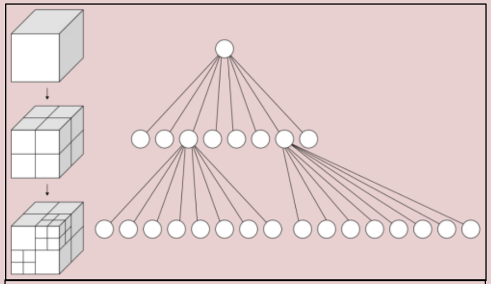
Collisional Simulations of Electron Cooling

- Accurate description of the cooling dynamics → **high precision direct numerical simulations** based on a minimal set of assumptions (consider the whole distribution) → **Collisional methods**.
- High accuracy leads to **efficiency challenges**:
  - Large particle numbers (but far from statistical limit)
  - Long range pair-wise interaction (both attractive and repelling)
  - Vast spatial and time scales
  - External electromagnetic fields
  - Maintain symplecticity

Efficient, Accurate Collisional Method: PHAD

Force computation $\mathcal{O}(N^2)$

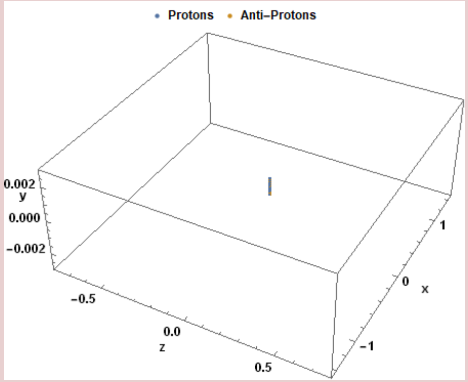
- **Reduced to $\mathcal{O}(N)$ using the FMM**
  - ➢ Adaptive hierarchical space decomposition



https://en.wikipedia.org/wiki/Octree

Accurate and efficient time stepping

- **Developed the Simo` Integrator**
  - ➢ Automatic selection of stepsize and order, optimal values according to Simo`'s theorem

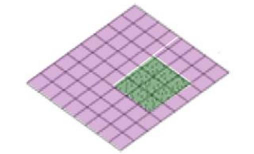


Long time-scale dynamics

- **Strang splitting**
  - ➢ Combine exact and numerical solutions
  - ➢ Maintain symplecticity

$$\frac{d\hat{Y}_j}{dt} = F_i(\hat{\mathbf{Y}}, t) = \underbrace{F_j^{[1]}(\hat{\mathbf{Y}}, t)}_{S_i} + \underbrace{F_i^{[2]}(\hat{\mathbf{Y}}, t)}_{S_i^c}$$

$$\hat{\mathbf{Y}}(h) = \phi^{[2]}_{h/2} \circ \phi^{[1]}_{h} \circ \phi^{[2]}_{h/2}(\hat{\mathbf{Y}}_0) + \mathcal{O}(h^3)$$

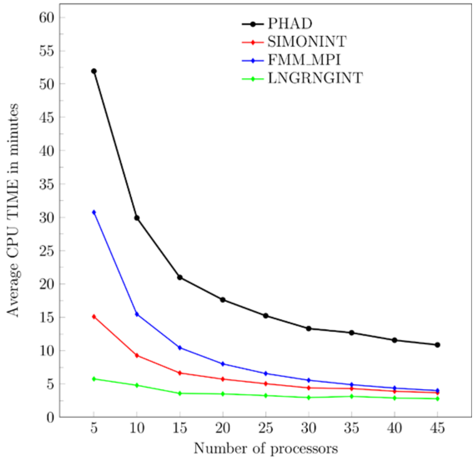


$$\dot{\mathrm{x}}_i = \frac{c\mathrm{p}_i}{\left(m_i^2c^2 + \|\mathrm{p}_i\|_2^2\right)^{1/2}} + 0$$

$$\dot{\mathrm{p}}_i = \frac{1}{4\pi\epsilon_0}\sum_{\substack{j\in N_i\\ j\neq i}} \mathrm{K}(\mathrm{x}_i, \mathrm{x}_j) + q_i\,(\mathrm{E} + \mathrm{B}\times \mathrm{v}_i) + \frac{1}{4\pi\epsilon_0}\sum_{j\in N_i^c} \mathrm{K}(\mathrm{x}_i, \mathrm{x}_j)$$

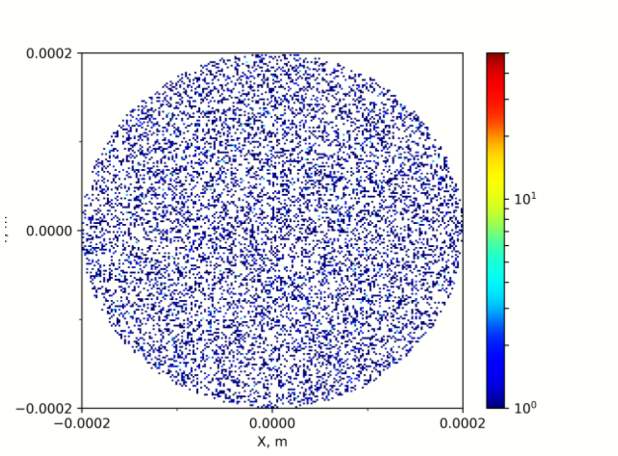
PHAD Features

- Accurate and efficient collisional simulation of charged particle beams in external electromagnetic fields **(minimal assumptions).**
- Fully adaptive both in space and time.
- Numerically symplectic over a long run.
- Best efficiency in reaching an a priori set error level.
- Ability to deal with very large N
  - Use distributed, high performance computing on hybrid architecture supercomputers

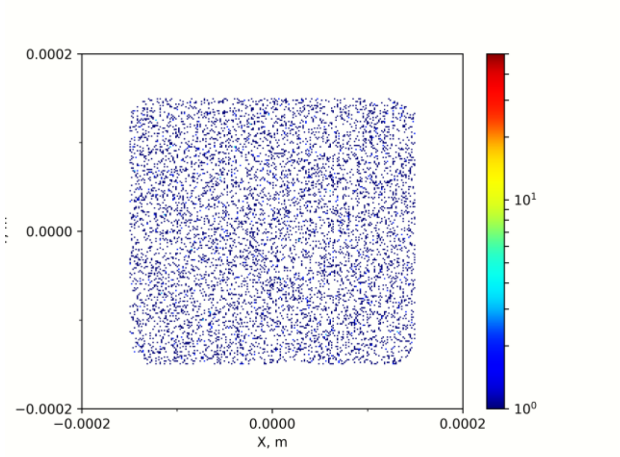


New Features since Last Meeting

- Window feature:
  - Represents a beamline’s boundaries
  - Eliminate particles that get to window’s limits



→



- Detailed beam loss map (in progress)

User Friendliness

- Website documentation and examples (in progress)- https://www.niu.edu/beam-physics-code/
- Parameter tracking - Simo integrator largest order and minimum timestep used
- Separate timers for read/write and some communications
- Descriptive error messages
- Relaunch capability
- Option to include a window and its limits

Electron Cooling Simulations

- PHAD will be the first code capable of providing a microscopic description of electron cooling with high fidelity, efficiently.
- **Benchmarking with experiments:**
  - Through collaboration with JLAB, simulations will be performed for the IMP's experiments
  - JLAB plans to cool ion beams using bunched electron beams → study ERL-based cooler to provide a highly bunched cooling electron beam with energy up to 55 MeV
  - To study such a cooling process, low energy cooling with bunched electron beams experiments were carried out at the storage ring CSRm at IMP in China

ZHANG, Yuhong – JLAB
BENSON, Stephen – JLAB
ZHANG, He – JLAB
WANG, Haipeng – JLAB
ZHAO, He – BNL

IMP Experiment

Xia, Jia-Wen, et al. "The heavy ion cooler-storage-ring project (HIRFL-CSR) at Lanzhou." *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 488.1-2 (2002): 11-25.

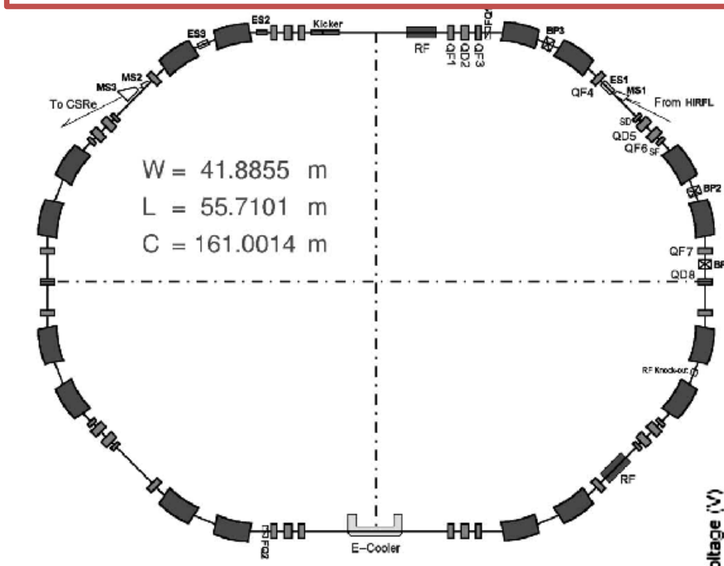


Fig. 4. Lattice layout of CSRm.

Zhao, H., et al. "Simulation of ion beam cooling with a pulsed electron beam." *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 902 (2018): 219-227.

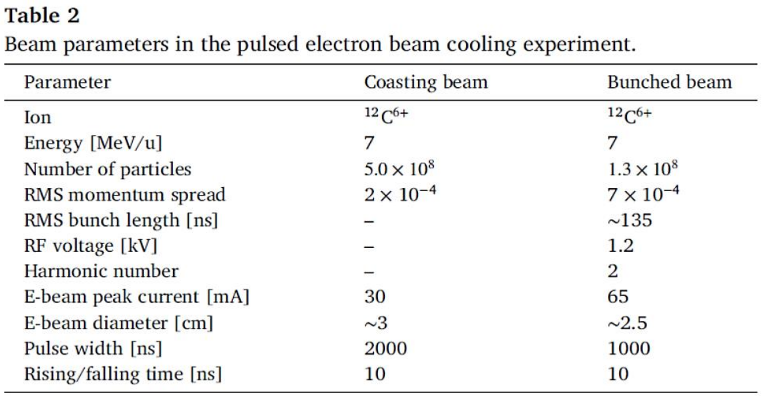
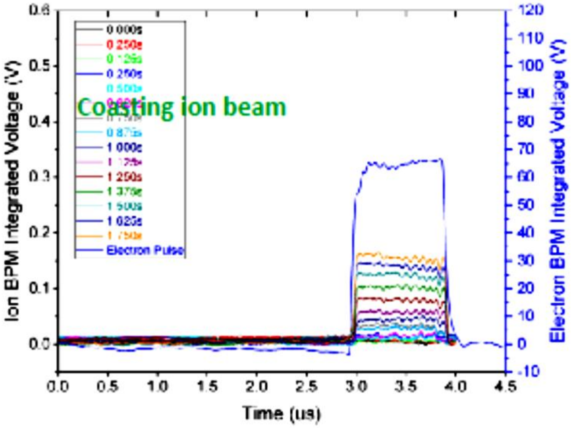
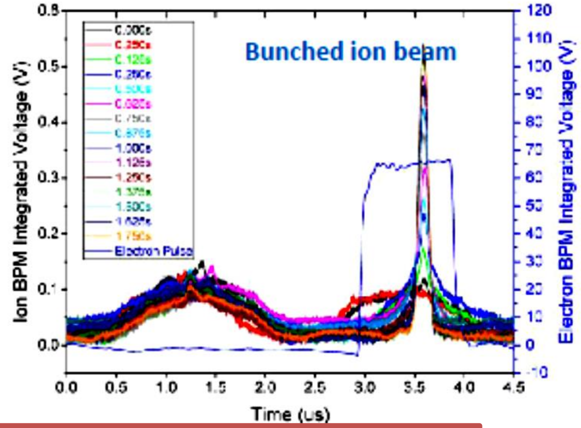


Table 2
Beam parameters in the pulsed electron beam cooling experiment.

Parameter	Coasting beam	Bunched beam
Ion	$^{12}C^{6+}$	$^{12}C^{6+}$
Energy [MeV/u]	7	7
Number of particles	5.0×10^8	1.3×10^8
RMS momentum spread	2×10^{-4}	7×10^{-4}
RMS bunch length [ns]	–	~135
RF voltage [kV]	–	1.2
Harmonic number	–	2
E-beam peak current [mA]	30	65
E-beam diameter [cm]	~3	~2.5
Pulse width [ns]	2000	1000
Rising/falling time [ns]	10	10

The 2nd cooling experiment run (2017)

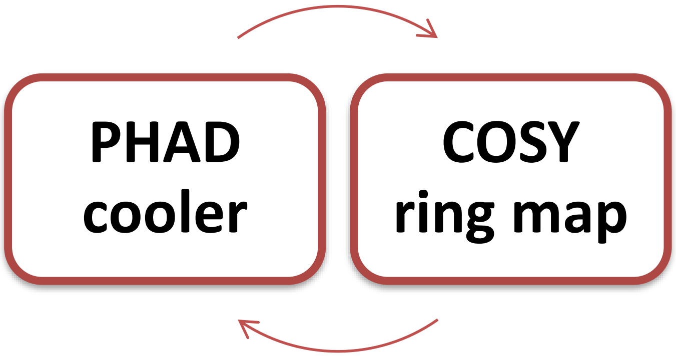




Yuhong Zhang and Haipeng Wang, “Bunched Beam Cooling Experiment.” *2018 Accelerator Research & Development PI Meeting,* November 13-14, 2018

Simulations Progress

- PHAD is an add-on to COSY INFINITY:
  - A general-purpose nonlinear beam dynamics code based on Differential Algebra
  - Large library of optical elements
  - Transfer maps of elements, and whole systems through composition of maps

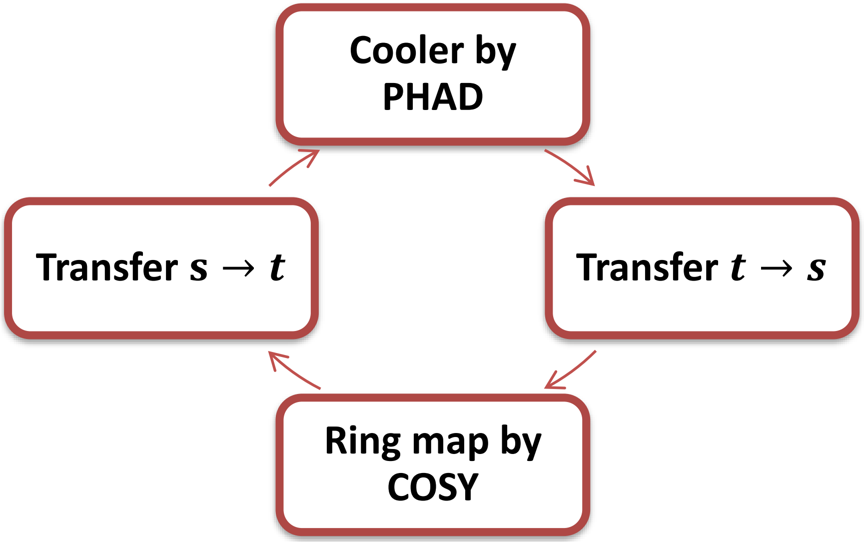


PHAD cooler

COSY ring map

Cooler Edges and Dependent Variable

- Ring map is s −dependent
- PHAD is t −dependent
- Keep transferring the dependent variable between s and t at exit/entrance of the cooler



Cooler by PHAD

Transfer $t \rightarrow s$

Ring map by COSY

Transfer $s \rightarrow t$

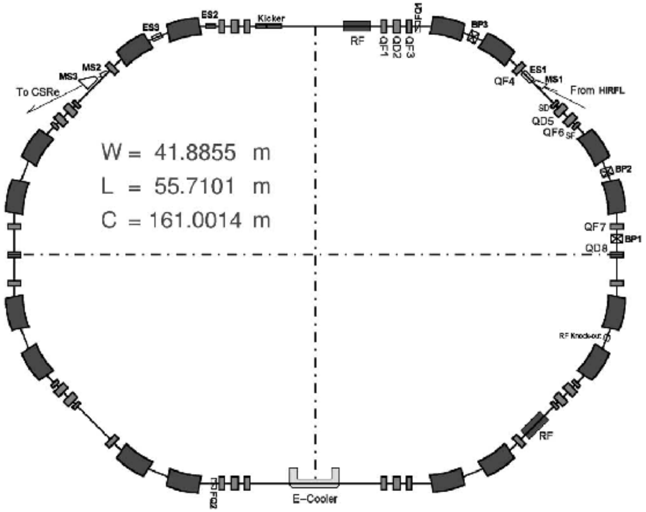
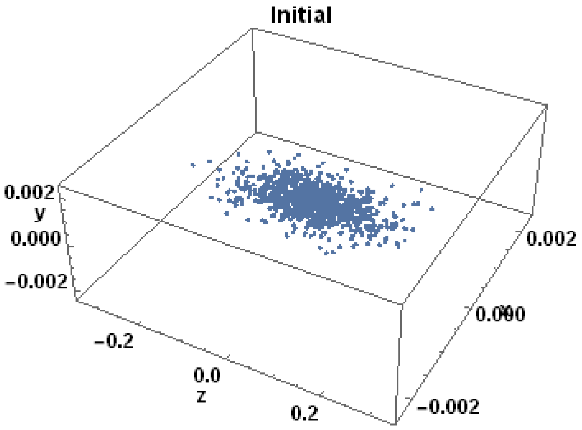
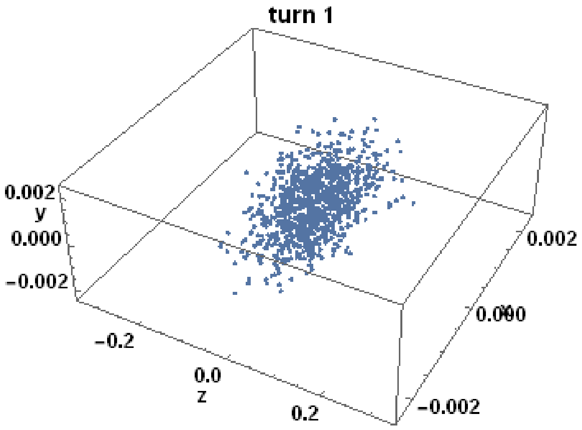


Fig. 4. Lattice layout of CSRm.

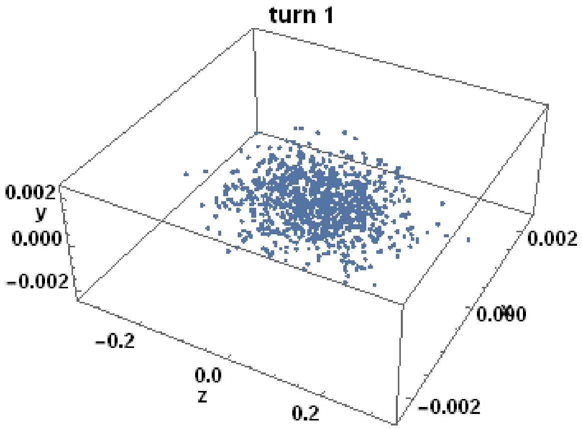
Initial Code Tests



Test 1:
Some code errors



Test 2:
Errors corrected



Modulator Section of CeC

- Main components of the CeC:

  Modulator → Amplifier → Kicker

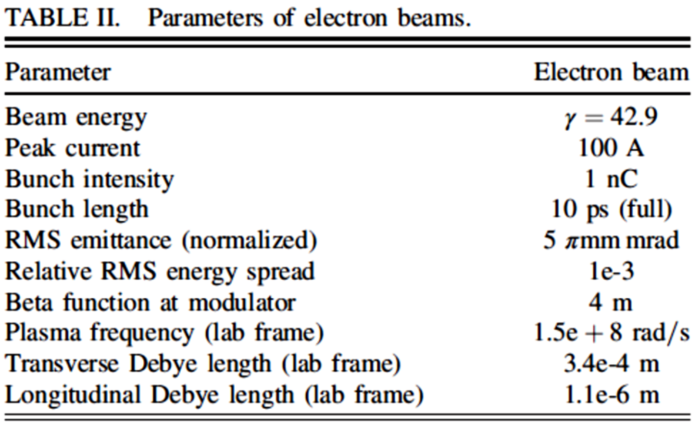


TABLE II. Parameters of electron beams.

Parameter	Electron beam
Beam energy	$\gamma = 42.9$
Peak current	100 A
Bunch intensity	1 nC
Bunch length	10 ps (full)
RMS emittance (normalized)	5 πmm mrad
Relative RMS energy spread	1e-3
Beta function at modulator	4 m
Plasma frequency (lab frame)	1.5e + 8 rad/s
Transverse Debye length (lab frame)	3.4e-4 m
Longitudinal Debye length (lab frame)	1.1e-6 m

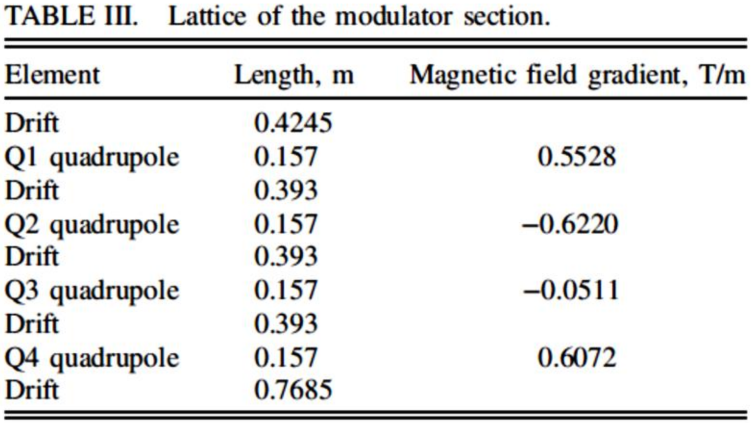


TABLE III. Lattice of the modulator section.

Element	Length, m	Magnetic field gradient, T/m
Drift	0.4245	
Q1 quadrupole	0.157	0.5528
Drift	0.393	
Q2 quadrupole	0.157	−0.6220
Drift	0.393	
Q3 quadrupole	0.157	−0.0511
Drift	0.393	
Q4 quadrupole	0.157	0.6072
Drift	0.7685	

Modulator section of PoP CeC experiments in RHIC at BNL

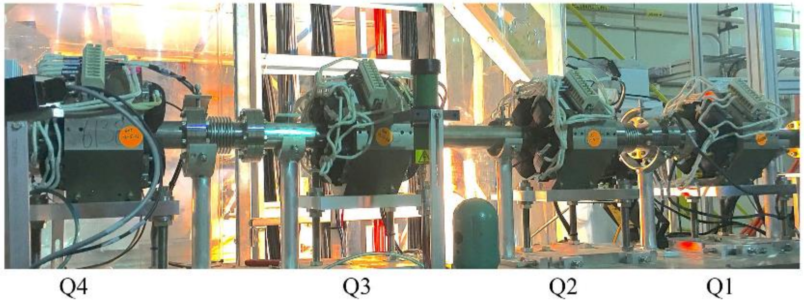
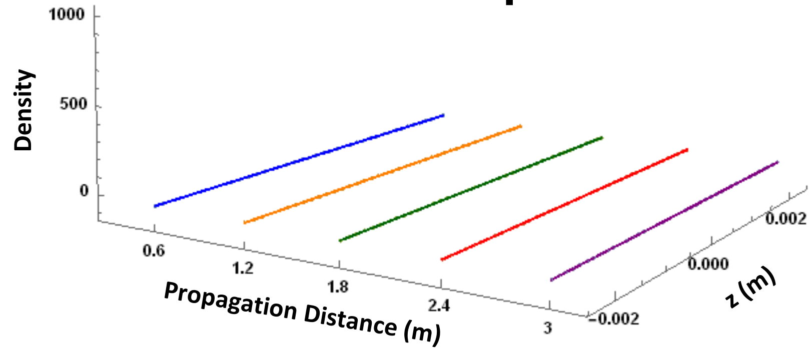


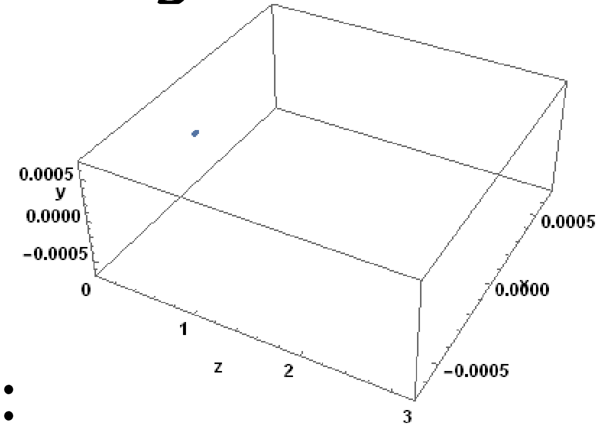
FIG. 10. CeC modulator section comprising four quadrupoles. The electron beam propagates from right to left.

Ma, J., Wang, X., Wang, G., Yu, K., Samulyak, R., & Litvinenko, V. (2018). Simulation studies of modulator for coherent electron cooling. *Physical Review Accelerators and Beams*, *21*(11), 111001.

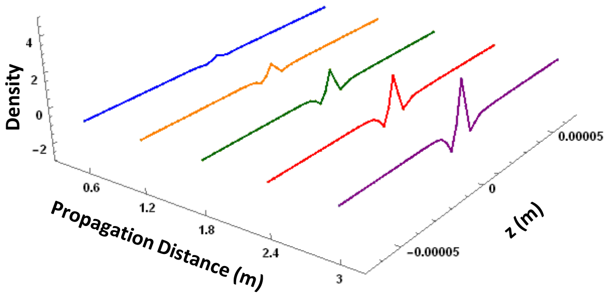
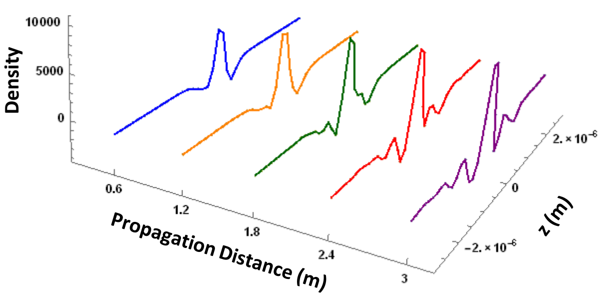
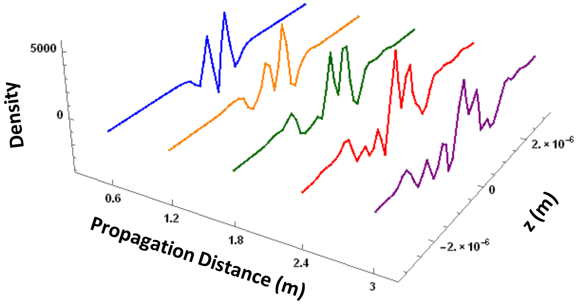
PHAD Simulations of the Modulator

- 10^5 electrons (BNL parameters) → no signal





- 10^4 electrons, different beam sizes:

100 µm	2 µm	2 µm – 2 ions
		

Summary

- Collisions play an important role in the process of electron cooling.
- We developed PHAD as a parallel N-body code that takes care of the collisions rigorously, with high fidelity, efficiently.
- PHAD will be the first code capable of particle-based simulations of electron cooling and other difficult beam dynamics phenomena.
- PHAD benchmarking with IMP’s experiments is in progress.
- PHAD simulations of modulator section of CeC provide microscopic details of density modulations.

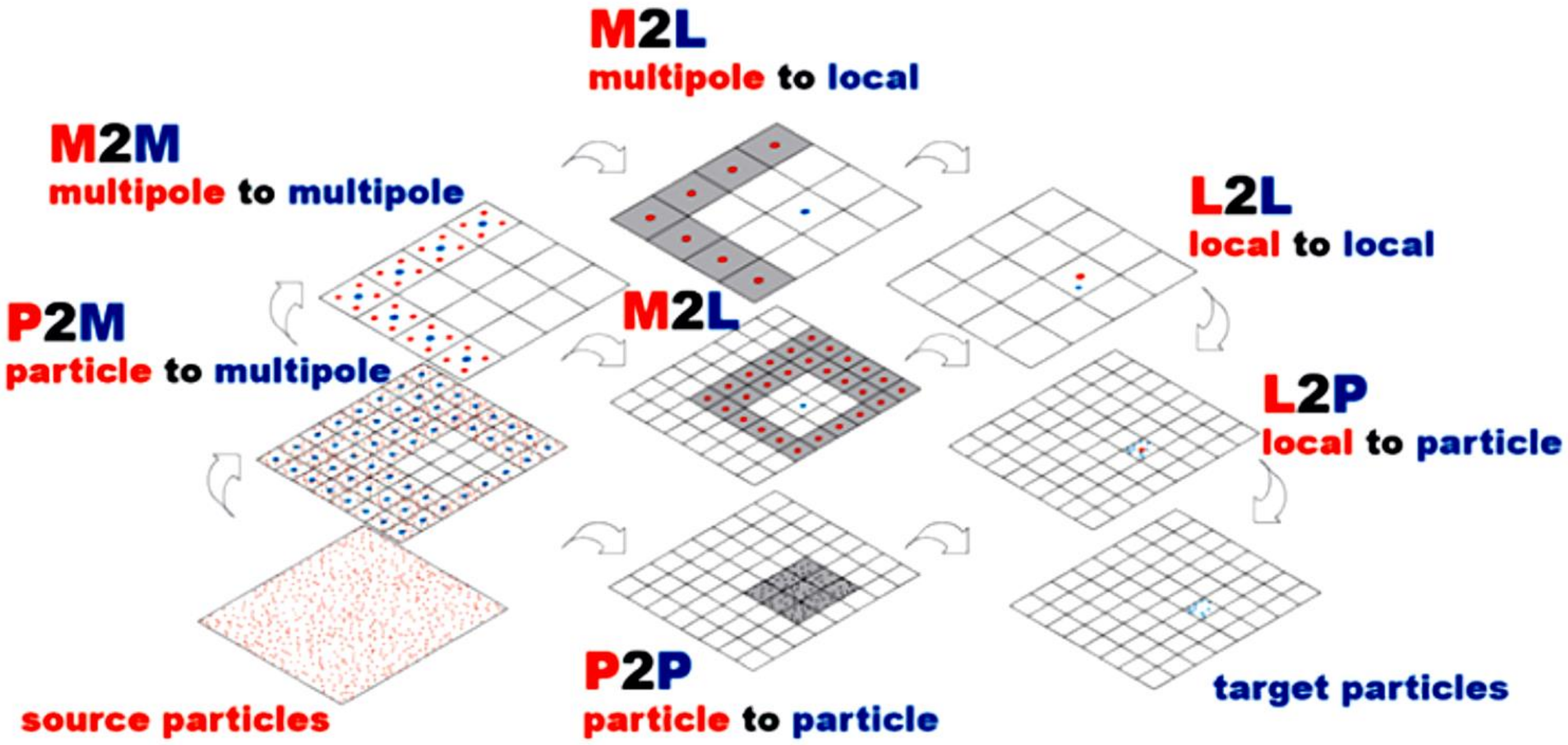
Website: https://www.niu.edu/beam-physics-code/

Thank You!

References

- S. Abeyratne, A. Gee, and B. Erdelyi, **An adaptive fast multipole method in Cartesian basis, enabled by algorithmic differentiation**, Communications in Nonlinear Science and Numerical Simulation 72 (2019): 294-317.
- H. D. Schaumburg, A. Al Marzouk, and B. Erdelyi, **Picard iteration-based variable-order integrator with dense output employing algorithmic differentiation**, Numerical Algorithms, (2018): 1-20.
- A. Al Marzouk and B. Erdelyi, **Collisional N-Body Numerical Integrator with Applications to Charged Particle Dynamics**, SIAM Journal on Scientific Computing 40.6 (2018): B1517-B1540.
- K. Makino and M. Berz, **COSY INFINITY version 9**, Nucl. Instr. Meth. Phys. Res., A, 558 (2005), pp. 346-350.
- S. Abeyratne, B. Erdelyi, and H. Schaumburg, **A novel code with high-order adaptive dynamics to solve the N-body problem**, (2015): MOPAB21.
- S. Abeyratne and B. Erdelyi, **N-body code to demonstrate electron cooling**, COOL2015, MOPF12

Fast Multipole Method



[Greengard and Rokhlin(1987)]. Diagram [Ibeid et al.(2016)]. (Siam top 10 algorithms of 20th century [Cipra(2000)])

Picard Integrator

- Picard iterations are used to prove existence and uniqueness of solutions of ODEs
- We implemented it in a Differential Algebraic framework to make it efficient numerically, and variable order to adjust for user requested accuracy
- Not adaptive, hence cannot handle well close encounters and more than one particle species

$$y' = ty^2, y(0) = 1.$$

$$y(t) = y(0) + \int_{t_0}^{t} s y(s)^2 ds$$

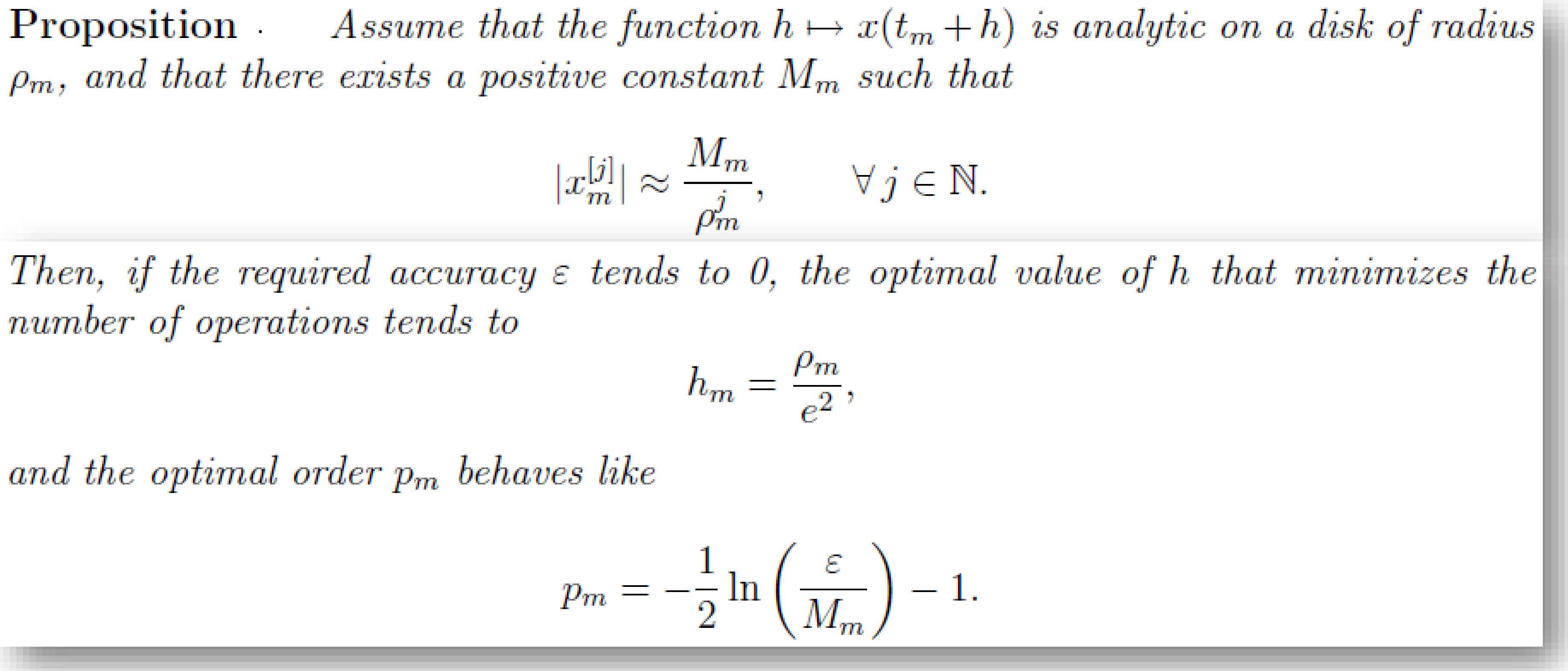
$$Y_0(t) = 1.$$

$$Y_1(t) = 1 + \int_{t_0}^{t} s(-1)^2 ds = 1 + \frac{1}{2}t^2.$$

$$Y_2(t) = 1 + \int_{t_0}^{t} s(-1 + \frac{1}{2}t^2)^2 ds = 1 + \frac{1}{2}t^2 - \frac{1}{4}t^3 + \frac{1}{24}t^6.$$

$$\begin{aligned} Y_3(t) &= 1 + \int_{t_0}^{t} s(1 + \frac{1}{2}t^2 - \frac{1}{4}t^3 + \frac{1}{24}t^6)^2 ds \\ &= 1 + \frac{1}{2}t^2 - \frac{1}{4}t^4 + \frac{1}{8}t^6 - \frac{1}{24}t^8 + \frac{1}{96}t^{10} - \frac{1}{576}t^{12} + \frac{1}{8064}t^{14} \end{aligned}$$

Optimal Order and Time Steps



Proposition . *Assume that the function* $h \mapsto x(t_m + h)$ *is analytic on a disk of radius* ρ_m*, and that there exists a positive constant* M_m *such that*

$$|x_m^{[j]}| \approx \frac{M_m}{\rho_m^j}, \qquad \forall\, j \in \mathbb{N}.$$

Then, if the required accuracy ε *tends to 0, the optimal value of* h *that minimizes the number of operations tends to*

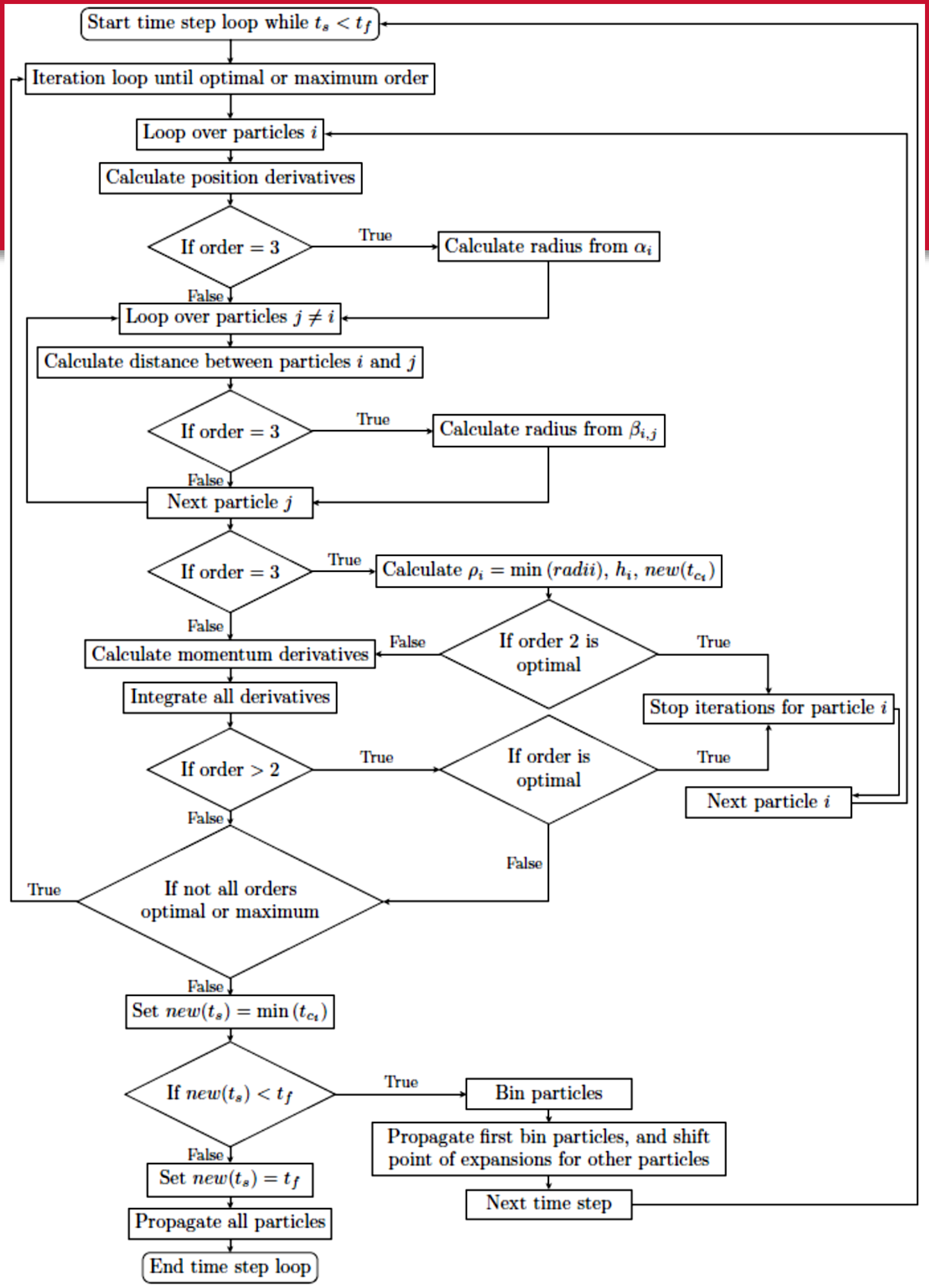
$$h_m = \frac{\rho_m}{e^2},$$

and the optimal order p_m *behaves like*

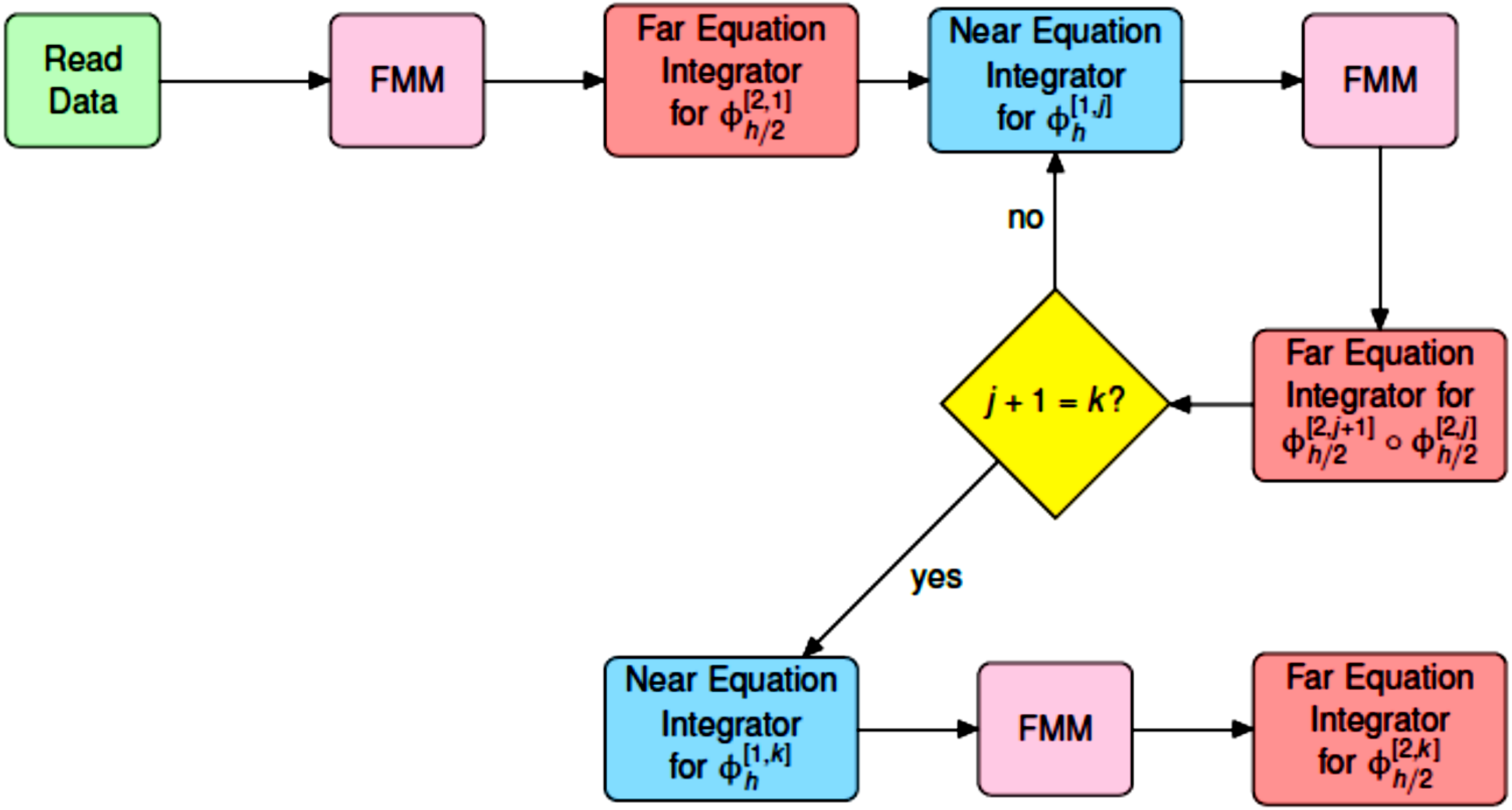
$$p_m = -\frac{1}{2}\ln\left(\frac{\varepsilon}{M_m}\right) - 1.$$

Simo (2001)

Simo` Integrator Flowchart



PHAD Algorithm Overview



Read Data

FMM

Far Equation Integrator for $\phi_{h/2}^{[2,1]}$

Near Equation Integrator for $\phi_h^{[1,j]}$

FMM

Far Equation Integrator for $\phi_{h/2}^{[2,j+1]} \circ \phi_{h/2}^{[2,j]}$

$j + 1 = k?$

no

yes

Near Equation Integrator for $\phi_h^{[1,k]}$

FMM

Far Equation Integrator for $\phi_{h/2}^{[2,k]}$

COSY INFINITY

- Precision dynamics modeling possible
  - Fringe fields
  - High order aberrations
- Possibility of inclusion of system parameters in the transfer map
- Extensive optimization methods readily available
  - Optimization loops available at the language level
  - 4 different optimizers
- Relative ease of adding new elements
- Fast
  - Optimization: optimize the map, no tracking involved
  - Tracking: just evaluate the map for many initial conditions

Elements in COSY

- Magnetic and electric multipoles
- Superimposed multipoles
- Combined function bending magnets with curved edges
- Electrostatic deflectors
- Wien filters
- Wigglers
- Solenoids, various field configurations
- 3 tube electrostatic round lens, various configurations
- Exact fringe fields to all of the above
- Fast fringe fields (SYSCA)
- General electromagnetic element (measured data)
- Glass lenses, mirrors, prisms with arbitrary surfaces
- Misalignments: position, angle, rotation

All can be computed to arbitrary order, and the dependence on any of their parameters can be computed.