Predictive Capabilities of the Dislocation-Network Theory of Harper–Dorn Creep

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The dislocation-network theory of Harper–Dorn (H–D) creep is reformulated using a new equation for the kinetics of growth of individual dislocation links in the network. The new kinetic equation has no impact on the scaled differential equation derived previously, which predicts the distribution of link lengths. However, the new theory predicts slightly different behavior for the kinetics of static recovery and leads to a new equation for the strain rate, which is expressed in terms of parameters that can be evaluated independently. This equation is valid not only for steady-state H–D creep, but is also valid for primary creep, provided the instantaneous value of the dislocation density is known. Using data on the variation of dislocation density with time, calculated values of the creep rates for Al deformed in the H–D regime agree with experimentally measured values to within a factor of 2. Creep curves for Al are calculated with the same degree of accuracy. These calculations involve no adjustable parameters. Steady-state creep rates for many materials presumably deformed in the H–D creep regime are compared with the predictions of the new equation for the strain rate. The calculated values agree with experimentally measured data to within a factor of about 150, which compares well with the predictions of other equations proposed in the literature.

I. INTRODUCTION

THE earliest attempts to explain the elevated-temperature mechanical behavior of crystalline materials on the basis of the dislocation-network theory[1–5] utilized average properties of the networks, primarily the dislocation density; these approaches were based on the early work of Frank.[6] The dislocation network itself consists of a three-dimensional array of individual dislocation lines (links) which meet at nodes; in general, there are three links per node. The dislocation density is the total length of all the links divided by the volume of the material. It is, therefore, an important averaged property of the network, but it tells us very little about the dynamics of network evolution. The early theories cited made no attempt to describe either the distribution of link lengths or the evolution of this distribution with time. This situation changed with the pioneering efforts of Lagneborg and his co-workers,[7–11] who made the first attempt to derive an equation, based on reasonable physical arguments, capable of describing the dislocation-link-length distribution and its relation to deformation. Not surprisingly, strain is produced by the motion of the dislocation links in the network in response to an applied stress. It is evident that the glide of individual links produces plastic strain, and it is certainly not difficult to visualize the idea that collisions between gliding links with each other or with momentarily stationary links can produce new nodes and refine the network. What is probably not so readily appreciated is the idea that positive plastic strain, *i.e.*, strain in the same direction as the applied stress, can also occur during *recovery* of the network. That is, plastic deformation can be accomplished through a *reduction* in the dislocation density, which is what happens when the network coarsens. We maintain that all the strain generated during Harper–Dorn (H–D) creep is a product of network coarsening.

The theory of Lagneborg and co-workers,[7–11] as well as our earliest efforts to explain high-temperature deformation using a dislocation-network model,[12] were flawed for a variety of reasons. Many of the deficiencies were rectified in our most recent article,[13] which incorporated several observations and ideas published after the appearance of the last article from the Swedish group. These ideas include scaling of the link-length distributions[14] and accounting for the separate contributions of freely gliding links and network links to the overall strain.[12,15] This latter idea leads, naturally, to an equation for the transition stress between power-law and H–D creep; this equation predicts the magnitude of this stress with no adjustable parameters. A critical look at network coarsening leads, naturally, to the idea of network frustration,[16] which helps explain how the dislocation density during H–D creep might be the same for different values of the applied stress.

In our most recent article,[13] we used the scaling behavior of the dislocation-link-length distribution, coupled with a kinetic equation for network coarsening, to predict strain rates and creep curves for comparison with existing data on H–D creep of Al. In this work, we revisit these ideas and incorporate previous work on the physics of the movement of dislocation nodes, to improve the equation for network coarsening. The improvement produces new results, wherein the creep rates and creep curves for Al during H–D creep are reproduced to a high degree of accuracy with no adjustable parameters. We also correlate extant data on steady-state

H–D creep with our equation for this process and compare the correlation with others proposed in the literature.

II. REVIEW OF THE THEORY

The distribution of link lengths is described by the function $\phi(L,t)$, where L is the length of an individual link in the network (which takes the form of a circular arc under applied stress σ), and $\phi(L,t)dL$ is the number of links per unit volume with a length between L and $L + dL$ at time t. The evolution of the dislocation network during H–D creep is described by the partial differential equation[13]

$$\frac{\partial\phi(L,t)}{\partial t} = -\frac{\partial\phi(L,t)g(L,t)}{\partial L} + A(t)\int_L^\infty \frac{\phi(L',t)}{L'}\,dL' \\ + B(t)\phi(L,t) \qquad [1]$$

where the constants $A(t)$ and $B(t)$ are given by the equations

$$A(t) = 2\,\frac{M_0(t)}{N} \qquad [2]$$

and

$$B(t) = \frac{1}{N}\left(\frac{dN}{dt} - M_0(t)\right) \qquad [3]$$

The important parameters N and $M_0(t)$ are the total number of links per unit volume and the rate of annihilation of links per unit volume, respectively, and are related to the link-length distribution by the equations

$$M_0(t) = -\lim_{L\to 0}(J(L,t)) \qquad [4]$$

and

$$N = \int_0^\infty \phi(L,t)dL \qquad [5]$$

where $J(L,t)$ is the flux of links leaving the interval L, $L + dL$ at time t. Equation [4] represents the fraction of links shrinking to zero length at time t. The flux is given by the equation

$$J(L,t) = \phi(L,t)g(L,t) \qquad [6]$$

where $g(L,t) \equiv dL/dt$ is the rate at which a link of length L changes with time.

One of the difficulties in the theory arises from determining an equation for $g(L,t)$. Lin *et al.*[14] showed that $\phi(L,t) \to L^m$, where m is a constant, as $L \to 0$ for all link-length distributions examined; the value of m is $\approx 4/3$ for Al and 3/2 for NaCl. This is an empirical result for which no theory exists, although it is consistent with the behavior of, for example, particle-size distributions during diffusion-controlled coarsening of precipitates ($m = 2$)[17] and grain growth ($m = 1$)[18] in the limit of vanishingly small sizes. Borrowing further from the theories of these processes,[17] $J(L,t)$ must remain finite in the limit $L \to 0$. This condition can be satisfied only if $g \to L^{-m}$ in the same limit. An equation for $g(L,t)$ which is consistent with these requirements is

$$g(L,t) = \frac{k'}{L^m}\left(\frac{L}{L^*(t)} - 1\right) \qquad [7]$$

where k' is a rate constant and $L^*(t)$ is a critical link length; links with length $L = L^*$ are momentarily stationary in time. The term in parentheses in Eq. [7] is also borrowed from coarsening theory and leads to a simple and reasonable equation which describes the kinetics of static recovery in terms of the time-dependence of the dislocation density.[19] Lagneborg also used Eq. [7] in his original article,[7] but with $m = 1$.

III. KINETICS OF LINK GROWTH

As mentioned earlier, Eq. [7] is largely empirical. It would be far more satisfactory if $g(L,t)$ were related in a more fundamental way to the dynamics of network growth. An equation which accomplishes this, and at the same time satisfies the constraints imposed by the distribution of link lengths, is proposed as follows. The starting point is the equation

$$g \equiv \frac{dL}{dt} = v_n\left(\frac{L^*}{L}\right)^m\left(\frac{L}{L^*} - 1\right) \qquad [8]$$

where v_n is the velocity of the dislocation nodes in the network. This equation normalizes L by a natural length scale of the system, namely L^*, and at the same time couples the coarsening of the network to the velocities of the nodes to which the individual links are attached.

Following the work of Evans and Knowles,[20] we assume that the nodes are spherical vacancy sinks/sources of diameter **b**, *i.e.*, equal to a Burgers vector, and that the climb velocity of a single node is equivalent to that of a jog. According to Hirth and Lothe,[21] the velocity of a jog is given by the equation

$$v_n = \frac{4\pi\mathbf{b}DF}{k_BT} \qquad [9]$$

where D is the lattice self-diffusion coefficient, k_B is Boltzmann's constant, and F is the force per unit length on the dislocation, which drives the motion of the node. In general, there are several contributions to F, including a contribution from the applied stress. However, when σ is small, as it is in the case of H–D creep, the main contribution to F originates from the net resolved force arising from the line tensions of the three dislocations at the node.[20] In this case, F, on average, is determined by the equation

$$F \approx \frac{\Gamma}{\langle L\rangle} \qquad [10]$$

where $\langle L\rangle$ is the length of the average link in the network and Γ is the dislocation-line tension. Substitution of Eq. [10] into [9] leads to the result

$$v_n = \frac{4\pi\mathbf{b}D\Gamma}{k_BT\langle L\rangle} \qquad [11]$$

Since the link growth rate will be proportional to the node velocity, the expression for g becomes

$$g(L,t) = \frac{dL}{dt} = \frac{4\pi\mathbf{b}D\Gamma}{k_BT\langle L\rangle}\left(\frac{L^*}{L}\right)^m\left(\frac{L}{L^*} - 1\right) \qquad [12]$$

A. *Scaling Equations*

We scale the individual values of L by L^* (t) and define the time-invariant measure of the link length (u) as

$$u = \frac{L}{L^*(t)} \qquad [13]$$

The scaled flux $(\vartheta(u))$ is defined as

$$\vartheta(u) = \Phi(u)G(u) \qquad [14]$$

where $\Phi(u)$ is the scaled link-length distribution function and $G(u)$ is defined by the equation

$$G(u) = \frac{u - 1}{u^m} \qquad [15]$$

The partial differential equation for $\vartheta(u)$ is

$$(u - 1)^2 \frac{d^2\vartheta(u)}{du^2} + M_{ou}u^m(u - 1)\frac{d\vartheta(u)}{du}$$
$$+ M_{ou}u^{m-1}((m + 1)(u - 1) - 1)\vartheta(u) = 0 \qquad [16]$$

which is identical in form to the one derived previously,[13] although M_{ou} is now given by the equation

$$M_{ou} = -\lim_{u \to 0} \Phi(u)G(u) = -\lim_{u \to 0} \vartheta(u) = \frac{(L^*)^2}{kN}M_0(t) \qquad [17]$$

where k is the rate constant for the kinetics of link-length coarsening, given by

$$k = \frac{4\pi \mathbf{b}D\Gamma}{k_B T\langle u \rangle} \qquad [18]$$

and $\langle u \rangle = \langle L \rangle / L^*$ is the average value of u. The two boundary conditions required to integrate Eq. [16] and find $\vartheta(u)$ and, subsequently, $\Phi(u)$ via Eq. [14] are $\vartheta(1) = 0$ and $\vartheta(0) = M_{ou}$.

B. *The Strain Rate*

In the dislocation-network theory of H–D creep, all the strain is due to dislocation-network coarsening, and the strain rate is given by relationship[12]

$$\varepsilon = \alpha \mathbf{b}r \int_0^{L_c} \phi(L,t)g(L,t)\sin^2\left(\frac{L}{2r}\right)dL \qquad [19]$$

where $\alpha \approx 0.5$ is a geometrical constant, and $r = 2\Gamma/\sigma\mathbf{b}$ is the radius of the bowed-out links under the applied stress, σ. The term L_c is equivalent to the length of a bowed-out dislocation segment during activation of a Frank–Read source. Expressions for the line tension Γ and L_c are standard and, ultimately, lead to an expression for the transition stress between H–D and power-law creep.[16] Using the dimensionless functions $\Phi(u)$ and $G(u)$, $L_c = \pi r$ and expanding $\sin^2(\pi u/2u_c)$ in a power series, taking advantage of the fact that $u \ll u_c$[13] and using the new equations for $g(L)$ and k in Eqs. [12] and [18], the strain-rate equation ultimately takes the form

$$\varepsilon = \frac{\pi C\mathbf{b}^3 D}{2k_B T}\rho\sigma \qquad [20]$$

where

$$C = \frac{\alpha}{\langle u \rangle^2}\int_0^{u_c} u^{2-m}(u - 1)\Phi(u)du \qquad [21]$$

C. *Network Coarsening*

An important assumption of the theory is that there is no fundamental difference between the kinetics of dislocation-network coarsening under applied stress and coarsening under stress-free conditions (*i.e.*, static recovery). The rate constant k, given by Eq. [18], is independent of stress under these conditions. On first writing Eq. [12] as

$$g(L,t) = \frac{dL}{dt} = \frac{k}{L^*}\frac{(u - 1)}{u^m} \qquad [22]$$

and taking advantage of the similarity between the mathematics of network coarsening and the coarsening of precipitates,[22] the growth of L^* during static recovery is readily shown to be governed by the relationship

$$(L^*)^2 - (L_0^*)^2 = \frac{2m^m}{(m + 1)^{m+1}}kt \qquad [23]$$

where L_0^* is the initial value of L^*. Taking advantage of the fact that $\langle u \rangle = \langle L \rangle / L^*$ and utilizing the relationship $\langle L \rangle = \beta/\rho^{1/2}$, Eq. [23] is readily rewritten as

$$\frac{1}{\rho} - \frac{1}{\rho_0} = \kappa t \qquad [24]$$

where ρ_0 is the initial dislocation density and κ is a rate constant given by the equation

$$\kappa = \left(\frac{\langle u \rangle}{\beta}\right)^2 \frac{2m^m}{(m + 1)^{m+1}}k \qquad [25]$$

Since the constants $\langle u \rangle$, β, and m are universal for a specific material and can be evaluated from data on the link-length distributions, Eq. [25] allows us to find κ from the analysis of data on static recovery. It is emphasized that Eq. [25] is expected to be valid primarily at the beginning of static recovery, because the network becomes frustrated on prolonged aging,[15] and there is no provision for that process in the theory. It is interesting to note that Eq. [24] is not new, but is identical to the type of equation that emerges from treating static recovery as a second-order kinetic process.[24] The derivation herein, which involves a mechanistic treatment of the kinetics of node movement and the phenomenology of the kinetics of network coarsening, provides an expression for the rate constant κ which has not been derived previously.

IV. COMPARISON WITH EXPERIMENTAL DATA

The viability of the theory is tested here in two different ways. The first is a comparison of its predictions with data on the experimental link-length distributions and strain rates relevant to the H–D creep of Al single crystals at 920 K under applied stresses of 0.05, 0.06, and 0.08 MPa.[13,14,15] The second evaluation of the theory involves a comparison between the steady-state creep rate predicted by Eq. [20] with all known experimental data on the steady-state creep

Table I. Compilation of the Data on the Link Length Distributions and Values of the Parameters Used in the Modeling of the H–D Creep of Al Single Crystals Tested in Compression at 920 K under the Stresses Indicated

σ (MPa) =	0.05	0.06	0.08	Average	St. Dev.	Validity
N (mm^{-3})	1045.5	987.4	878.8	970.5	84.6	steady state
$\langle L \rangle$ (mm)	0.0826	0.0841	0.0872	0.0846	0.0024	
ρ (mm^{-2})	86.32	83.00	76.61	81.98	4.98	
L^* (mm)	0.0403	0.0400	0.0423	0.0408	0.0012	
$\langle u \rangle$	2.058	2.108	2.068	2.078	0.0273	primary and steady-state region
β	0.767	0.766	0.763	0.7653	0.0021	
M_{ou}	0.556	0.509	0.611	0.559	0.051	
κ (m^{2}s^{-1})	—	—	—	1.918×10^{-9}		
ψ (mm^2)	8.809×10^{-3}	—	1.169×10^{-2}	1.025×10^{-2}	2.035×10^{-3}	
τ (s)	2.879×10^5	—	8.748×10^5	5.81×10^5	4.15×10^5	
C	0.264	0.266	0.270	0.267	0.003	

rates during H–D creep. This comparison is made possible by Eq. [20], which provides the dependence of ε_s on material parameters.

A. *The H–D Creep of Aluminum*

This comparison is identical in many ways to the one the authors previously published,[13] but it now takes into account the newly developed relationship for the kinetics of network coarsening. Since the differential Eq. [17] is unaffected by the different forms of Eqs. [7] and [12], there is no change at all of the parameters derived from the distributions themselves, *i.e.*, N, $\langle L \rangle$, ρ, L^*, $\langle u \rangle$, **b**, and M_{ou} are exactly the same as they were before, although C is different because of the way it has been defined in Eq. [21]. The values of these parameters are shown in Table I.

We evaluate κ as we did previously, *i.e.*, by examining the data of Hausselt and Blum[25] on static annealing of the dislocation microstructures in an Al-11 pct Zn alloy after deformation in the steady-state creep regime at 523 K under three different stresses. Their data are shown in Figure 1(a), and the data on the shorter annealing times are plotted as ρ^{-1} *vs* t in Figure 1(b), where it is seen that κ, which is obtained from the slopes of the curves in Figure 1(b), is nearly constant for all three stresses. The average value of κ estimated from the slopes of the least-squares regression lines is $2.226 \pm 0.488 \times 10^{-15}$ m^2 s^{-1}. Since this value of κ pertains to static recovery at 523 K, it is necessary to estimate its value at 920 K, which is the temperature used for the creep tests on Al.[15] Using the temperature dependence of k and κ on D/T from Eqs. [18] and [25] and the activation energy of 143.4 kJ/mol for self-diffusion in Al,[26] we find that $\kappa = 1.918 \times 10^{-9}$ m^2 s^{-1} at 920 K.

Comparisons of the experimental and predicted creep rates are shown in Figures 2(a) through (c). Figure 2(a) shows the steady-state creep rates *vs* applied stress, Figure 2(b) shows predicted and experimentally measured creep rates during primary creep *vs* time for samples tested at 0.05 MPa, and Figure 2(c) shows the primary-creep data for samples tested at 0.08 MPa. The data in Figures 2(b) and (c) were taken from the work of Lee.[23] In all cases, the theory predicts systematically smaller creep rates than the experimentally measured ones by a factor of approximately 2. We regard the agreement between theory and experiment as outstanding, considering that primary and steady-state creep rates have been predicted from the microstructural data alone with *no adjustable parameters*.
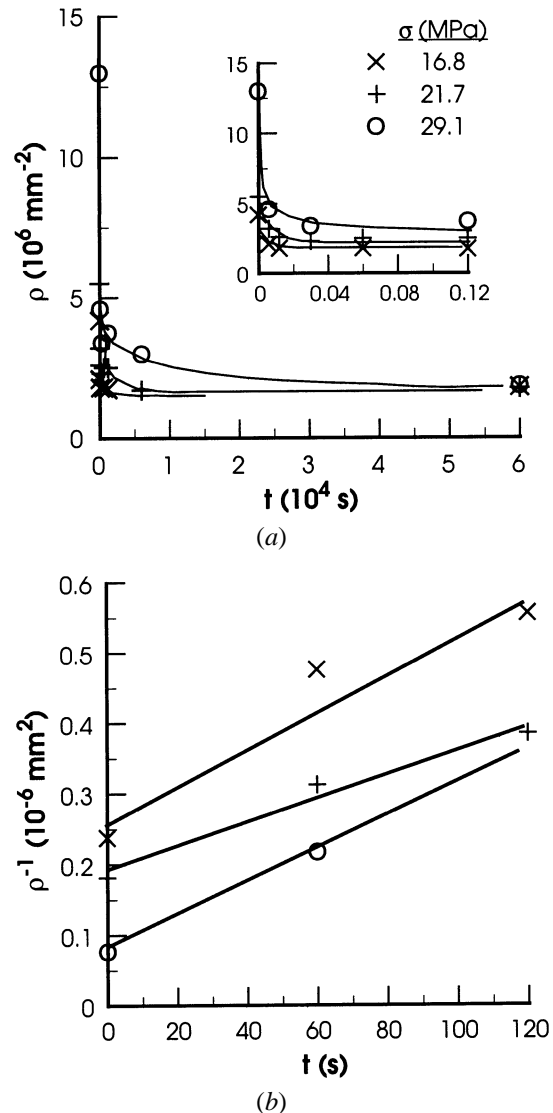

(a)

(b)

Fig. 1—(a) Long- and (b) short-time data of Hausselt and Blum[25] on dislocation recovery under zero stress in an Al-11 pct Zn alloy deformed at 523 K. The short-time data are plotted in (b) as ρ^{-1} *vs* t to obtain values of the dislocation link-growth-rate constant κ.

Equation [20] can also be integrated to obtained the creep curves, provided that the change of dislocation density with time is known. The variation of ρ with t, using the data
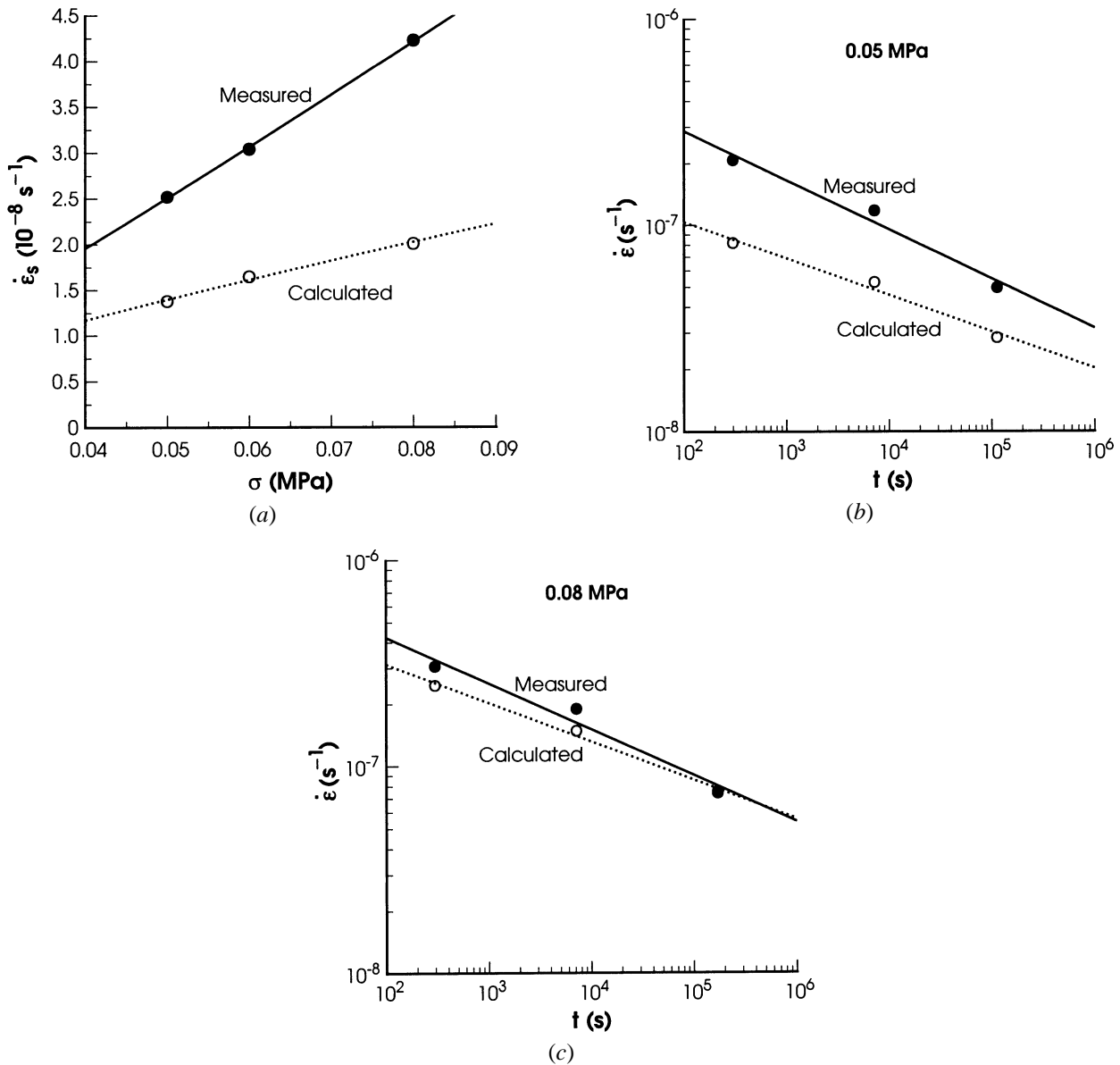
Fig. 2—Comparison of the measured and predicted creep rates for steady-state and primary creep *vs* time for Al single crystals tested at 920 K: (*a*) steady-state creep, (*b*) primary creep under an applied stress of 0.05 MPa, and (*c*) as in (*b*) but for 0.08 MPa.

measured by Lee,[23] is shown in Figure 3(a). We assume that the dislocation density during primary creep is governed by the equation

$$\rho_s^{-1} - \rho^{-1} = (\rho_s^{-1} - \rho_i^{-1})\, e^{-t/\tau} \qquad [26]$$

where ρ_i is the dislocation density on loading the specimen and τ is a phenomenological constant. Accordingly, the data of Lee[23] corresponding to the primary-creep regime are shown in Figure 3(b), plotted in the form ln (ρ_s^{-1} − ρ^{-1}) *vs* t. The straight lines providing the best fit with the data are shown, and it is evident that Eq. [26] does not describe the data very accurately. It is, nevertheless, good enough for our purposes and provides the estimates of ρ_i and τ for the creep tests done at 0.05 and 0.08 MPa. Combining Eqs. [20] and [27] with the values of τ and ψ shown

in Table I and solving for ε gives the following expression for the creep curve:

$$\varepsilon = \frac{C\pi \mathbf{b}^3 D}{2k_B T}\, \tau\sigma\rho_s \left(\frac{t}{\tau} + \ell\mathrm{n}\left(\frac{1 - \psi\rho_s\, e^{-t/\tau}}{1 - \psi\rho_s} \right) \right) \qquad [27]$$

where $\psi = \rho_s^{-1} - \rho_i^{-1}$.

The creep curves calculated using Eq. [24] are in quite good agreement with the two experimental curves, as can be seen in Figure 4. There are no adjustable parameters. The calculated values of $\dot{\varepsilon}_s$ are smaller than those measured experimentally, which manifests itself graphically in the smaller steady-state creep rates at both applied stresses. It is perhaps worth noting that the value of κ predicted by Eqs. [18] and [25] is about a factor of 3 smaller than the average value of κ in Table I, which was obtained from analysis of the data of Hausselt and Blum.[25]
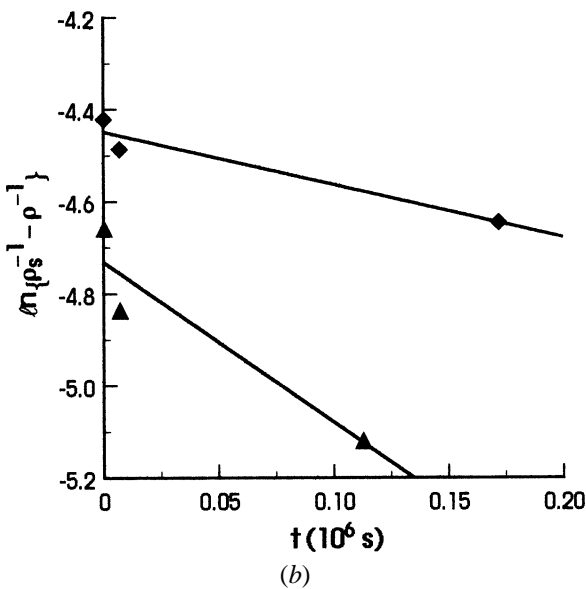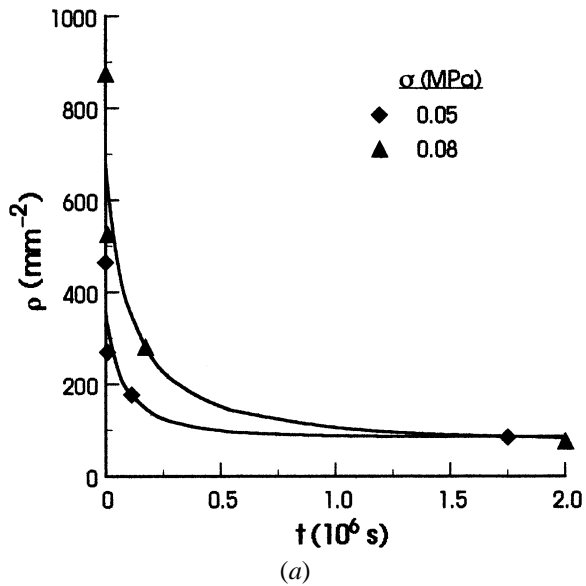
Fig. 3—(a) The experimental data of Lee[23] on the change of the dislocation density vs time during primary H–D creep of Al single crystals. The data in (a) are replotted in (b) as ln $\{\rho_s^{-1} - \rho^{-1}\}$ vs t to obtain the phenomenological constants ψ and τ (Eqs. [26] and [27]).
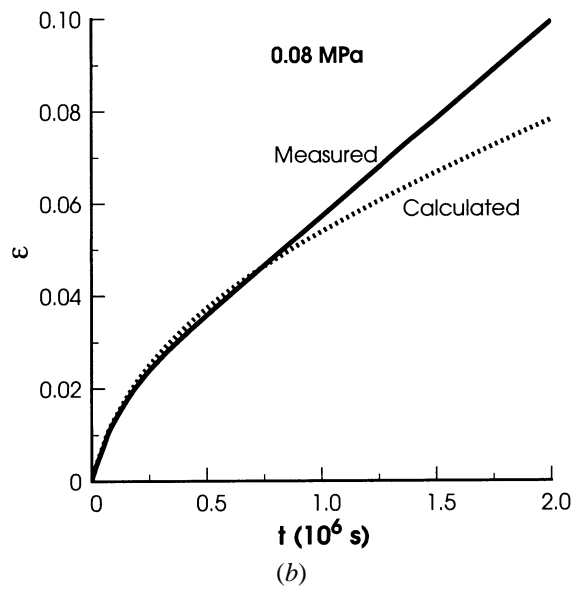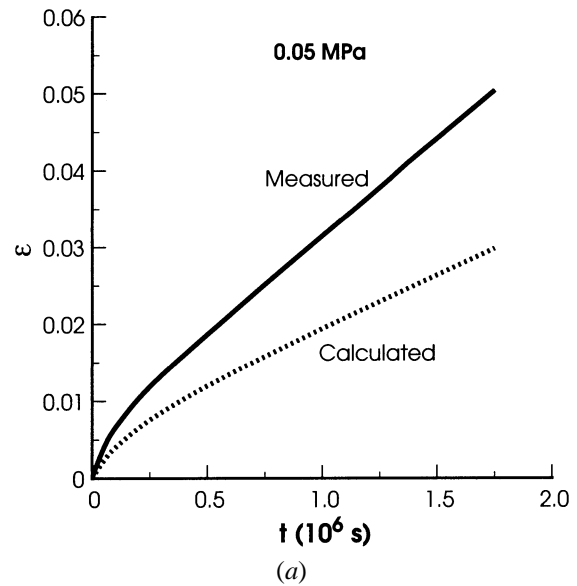
Fig. 4—Comparison of the experimental and predicted creep curves for monocrystalline Al tested in compression at 920 K under applied stresses of (a) σ = 0.05 MPa and (b) σ = 0.08 MPa. The experimental curves are from the data of Lee.[23]

B. Steady-State Creep-Rate Correlations for H–D Creep

Equation [20] can be rearranged in dimensionless form to give

$$\frac{2k_B T \dot{\varepsilon}_s}{\pi C \mathbf{b}^3 \rho D G} = \frac{\sigma}{G} \qquad [28]$$

where the shear modulus (G) has been introduced artificially, as it often is, for dimensional purposes.

If we regard the parameters from the distribution of link lengths as universal, i.e. assuming that the constant C is the same for all materials, we can plot all the known data on H–D creep to test the validity of Eq. [28]. Such a plot is shown in Figure 5, where it is evident that the data are scattered around the line of perfect agreement to within a factor of about 150. In constructing this plot, we have used only those data which were clearly from the H–D region

and where the authors included values of the dislocation density. All of the points, with the exception of the few data points for the highest stresses for olivine, were within our theoretical limit for H–D creep.[16] This limit requires that the acting stress be smaller than the stress necessary to bow the largest links in the distribution (equal to $\sim 3\langle L \rangle$) into a semicircular arc. One factor contributing to the discrepancy in the case of olivine is the absence of reliable data on dislocation densities, which are difficult to measure in this material. Another possible source of uncertainty is the magnitude of the constant C, which was evaluated from Eq. [21]. It should be kept in mind that many of the materials involved in the correlation in Figure 5 have limited numbers of slip systems, even at elevated temperatures, and might, therefore, have rather different dislocation-link-length distributions than Al. This would certainly have some influence on the calculation of C, although we doubt that its magnitude would
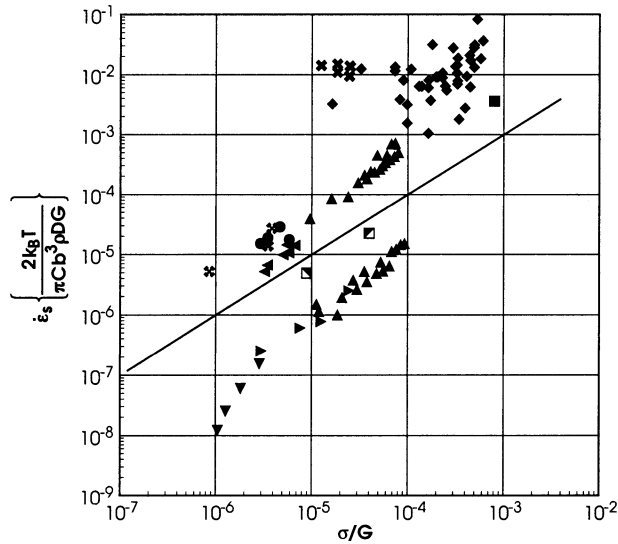
Fig. 5—Comparison of the available experimental data on steady-state H–D creep with the predictions of the dislocation network theory, Eq. [28]. As discussed in the text, only the data for which both the creep rates and the dislocations densities were known could be used in the comparison. The sources of data are summarized in Table II.

Table II. References Used as the Sources of Data for Estimating the Normalized Strain Rates in Figures 5 through 7; the Two Sets of Data on Al Were Obtained from Monocrystalline and Polycrystalline Specimens

Data Set	Strain Rates and Stresses	Dislocation Density	Diffusion Coefficient	Plotting Symbol
Al (mono)	23	23	33	●
Al (poly)	34	34	33	✕
NaCl	35	35	33	▲
α-Zr	36	36	37	▲
MgO	38	39	33	✕
Fe-3Si	40	41	33	▼
Al-5Mg	42	42	33	▶
Quartz	29	29	12	■
α-Ti	43	43	33	◩
β-Co	43	43	13	◪
Olivine	44, 45	44, 45	33	◆

be affected by as much as a factor of 10. Nevertheless, there is no reason to think that C would be identical for materials with very different crystal structures and slip systems. Finally, we have made no attempt to distinguish between lattice diffusion and dislocation pipe diffusion, even though it is suspected that pipe diffusion could play an important role in H–D creep at lower temperatures.[27]

It is useful to compare the correlation in Figure 5 with those obtained from other equations for H–D creep. To this end, we have chosen the equations proposed by Langdon and Wang,[28] Wang,[29] and Ruano et al.[30] Langdon and Wang assumed that the H–D creep rate is given by the equation

$$\dot{\varepsilon} = \rho \mathbf{b} V_c \qquad [29]$$

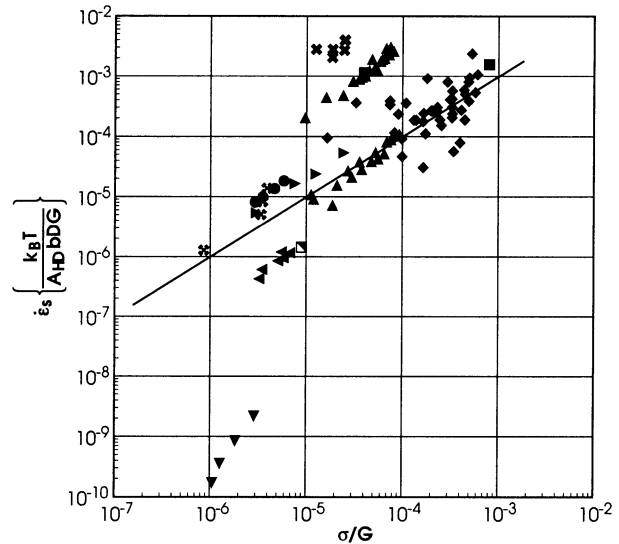where V_c is the climb velocity of jogged dislocations, which

Fig. 6—Same data as in Fig. 5, used for the comparison with the predictions of the equation for the steady-state creep rate, Eq. [31], proposed by Langdon and Wang.[28,29]

is assumed to be controlled by the lattice diffusion of vacancies under saturation conditions. Based on theoretical considerations, they proposed that the steady-state dislocation density during H–D creep is related to the Peierls stress (τ_p) through the relationship

$$\rho^{1/2} = \frac{1.3\tau_p}{G\mathbf{b}} \qquad [30]$$

The use of a reasonable expression for V_c in conjunction with Eqs. [29] and [30] leads to their equation for the normalized strain rate, i.e.,

$$\frac{\dot{\varepsilon}k_B T}{A_{HD} D_L G \mathbf{b}} = \frac{\sigma}{G} \qquad [31]$$

where

$$A_{HD} = 1.4 \left(\frac{\tau_p}{G}\right)^2 \qquad [32]$$

$$\tau_p = \frac{G}{1-v} \exp\left(-\frac{2\pi}{(1-v)}\frac{d}{\mathbf{b}}\right) \qquad [33]$$

and d is the slip-plane spacing. The data in Figure 5 are plotted in Figure 6 in accordance with the functional form of Eq. [31].

Ruano et al.[30,31,32] proposed a normalization based on an approach involving an internal stress (σ_i). Their general expression for the creep rate has the form

$$\dot{\varepsilon} = \frac{1}{2} A_{PL} \frac{D_{eff}}{\mathbf{b}^2} \left(\left(\frac{\sigma + \sigma_i}{E}\right)^n \right.$$
$$\left. + \frac{|\sigma - \sigma_i|}{\sigma - \sigma_i} \left|\frac{\sigma - \sigma_i}{E}\right|^n \right) \qquad [34]$$

where the stress exponent $n = 5$ is appropriate for power-law creep; D_{eff} is an "effective" diffusion coefficient, which for H–D creep is equivalent to D,[30] and A_{PL} is an empirical material constant. For small stresses in the H–D regime where $\sigma < \sigma_i$, and with σ_i given by the equation
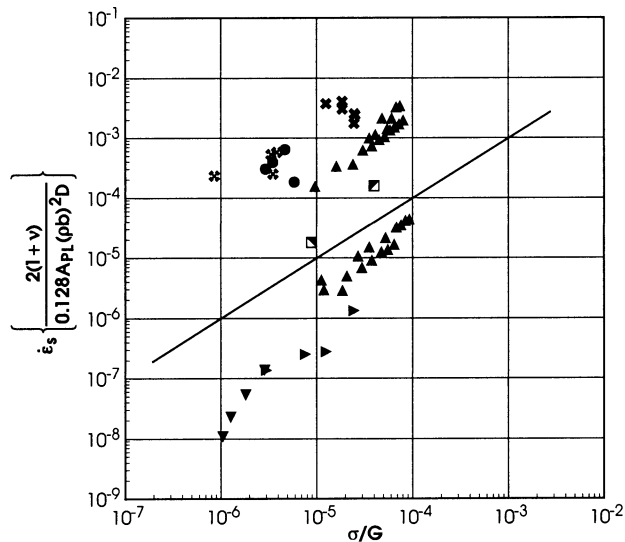
Fig. 7—Comparison of the predictions of the steady-state creep rate equation of Ruano *et al.*, Eq. [36], with experimental data. The data used were the same as those in Figs. 5 and 6, but the values of the constant A_{PL} were not calculated by Ruano *et al.* for all the materials.

$$\frac{\sigma_i}{E} = 0.4\mathbf{b}\sqrt{\rho} \qquad [35]$$

the equation of Ruano *et al.* in normalized form is

$$\frac{2(1 + v)\varepsilon}{0.128 \, A_{PL}(\mathbf{b}\rho)^2 D_{\mathrm{eff}}} = \frac{\sigma}{G} \qquad [36]$$

The data in Figures 5 and 6 are plotted in Figure 7 according to the functional form of Eq. [36]. Some of the data in the previous figures are omitted, because Ruano *et al.* did not analyze the data on those materials and, therefore, did not determine values of A_{PL} for them.

Equation [31] of Langdon and Wang[28,29] describes the data about as accurately as Eq. [28], but Eq. [36] of Ruano *et al.*[30,31,32] does not fare as well, even though their reported values of A_{PL}, obtained from best fits of their analyzed data, were used in the correlation in Figure 7. Given that the role of dislocation pipe diffusion is probably more important in some systems under specific conditions of testing than in others, and that the dislocation densities do not appear to have been measured carefully by some of the investigators, the correlation is about as good as might be expected.

V. SUMMARY

The dislocation-network theory of H-D creep has been updated through the use of a more realistic equation for describing the kinetics of growth of dislocation links, *i.e.*, Eq. [12]. This equation incorporates physical factors involved in the motion of the nodes in the network and is, therefore, on firmer physical ground than the rate equations used previously, even though it still includes a semiempirical factor for consistency with the experimentally measured link-length distributions and a phenomenological factor borrowed from the theory of particle coarsening. The analysis of data on the kinetics of static recovery in an Al-Zn alloy yields a value of the rate constant which can be substituted directly into the equation for the strain rate during H–D creep. This

substitution allows us to calculate the instantaneous and steady-state creep rates, and the creep curves, with very good accuracy. The theoretical equation for the strain rate is in fair agreement with a large body of data in the literature on steady-state creep in the H–D regime. It is certainly as good as some others, though it is far from perfect.

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